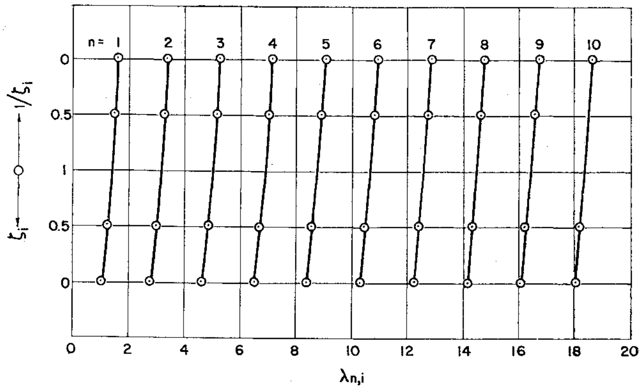

Fig. 1 Variation of eigenvalues with ζ_i.

Equation (5) may be used to evaluate the $A_{n,i}$ coefficients in the manner customary with orthogonal functions. The only restriction on $Y_{i,0}(\eta)$ is that it must decay exponentially at least as fast as $[(\eta - \varkappa)^2/4]$ as $\eta \to \infty$.

In principle, the solution given by Eqs. (2-4) is exact so that it is possible to compute, for example, the distribution of species concentration downstream of a discontinuity in ζ_i. This problem for the limiting case of infinite Schmidt number has been solved in Ref. 2. Here, of course, the Schmidt number is unity. If the perturbation point of view of Refs. 1 and 5 is followed, there can be computed, with little effort, first- and higher-order corrections to the present solution for deviations of the velocity field from that described by the Blasius function, for variations in the $\rho\mu$ product, and for nonunity Schmidt number, provided that the surface catalyticity is indeed constant.

The most convenient and accurate method of determining the eigenvalues and eigenfunctions defined by Eqs. (3) and (4) has been found to involve use of the asymptotic solution [which applies to Eq. (3)], i.e., the solution when $f_0 \simeq \eta - \varkappa$, $f_0' \simeq 1$, and in which the function with power law decay in the form $(\eta - \varkappa)^{-\lambda_{n,i}}$ is suppressed. By picking arbitrarily a value of $N_{n,i}$ at $\eta = \eta^*$, where η^* satisfies the inequality $|1 - \lambda_{n,i}|(\eta^* - \varkappa)^{-2} \ll 1$, and a value of $\lambda_{n,i}$, the value of $N_n{}',{}_i(\eta^*)$ on the asymptotic solution for $\eta > \eta^*$ may be computed, and the integration in the direction of decreasing η can be carried out. When $\eta = 0$ is reached, the value of $\zeta_i = N_{n,i}'(0)/N_{n,i}(0)$ is obtained. If a particular value of ζ_i is desired, a new guess for $\lambda_{n,i}$ must be made and the integration repeated. On the contrary, if an eigenvalue for a particular value of ζ_i is available, then the integration can proceed from the wall ($\eta = 0$) outward with either $N_{n,i}(0) = 1$ or $N'_{n,i}(0) = 1$, for example, and the eigenfunction and value of the normalizing parameter $C_{n,i}$ obtained from a single integration and quadrature. Clearly, a small scale computer is adequate for this work.

The first ten eigenvalues for $\zeta_i = 0, \infty$ given in Ref. 1 have been supplemented in the present work by the corresponding first ten values for $\zeta_i = \frac{1}{2}$ and the first nine values of $\zeta_i = 4$; these results are presented numerically in Table 1 and graphically in Fig. 1. The significant figures in Table 1 reflect the accuracy with which the specified values of ζ_i have been achieved. The results presented here permit ready estimates to be made for the eigenvalues for any value of ζ_i and for values of $n > 10$; this is suggested by the "smoothness" of the curves of $\lambda_{n,i}$ vs ζ_i for a given n and by the "almost equal" spacing of successive $\lambda_{n,i}$'s, namely, a spacing of 1.94.

Table 1 Eigenvalues for various surface catalyticity

$n \backslash \zeta_i$	0	$\frac{1}{2}$	4	∞
1	1	1.26	1.50	1.573
2	2.77	3.00	3.29	3.385
3	4.62	4.83	5.14	5.25
4	6.51	6.70	7.02	7.14
5	8.41	8.59	8.92	9.05
6	10.32	10.50	10.82	10.96
7	12.24	12.41	12.74	12.88
8	14.17	14.33	14.66	14.81
9	16.10	16.26	16.59	16.74
10	18.04	18.19	...	18.68

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Reduction of Torsional Stiffness Due to Thermal Stress in Thin, Solid Wings

D. J. Johns*

Loughborough College of Technology, Loughborough, Leics, England

RESULTS are given in Ref. 1 which show how the spanwise thermal stresses caused by aerodynamic heating can produce a reduction in the torsional stiffness of thin wing sections. In Ref. 2 the analyses have been extended to include the effects of cross-sectional shape, solidity, and other parameters. The purpose of the present note is to examine the implications of using assumed temperature distributions in such analyses and to determine whether these distributions can be readily correlated with the temperatures that would result at only three points on the wing semichord for an actual aerodynamic heating problem. The notation used and assumptions made are similar to those of Ref. 1.

Consider the symmetric solid section (Fig. 1) whose chordwise thickness variation is given by

$$t = t_0 + \Delta t|\beta|^m \tag{1}$$

and let the chordwise temperature rise distribution be approximated by

$$T = T_0 + \Delta T|\beta|^n \tag{2}$$

The spanwise thermal stresses are determined from the simple equation

$$\sigma_{yy} = E\alpha[\textstyle\int_A T dA/A - T] \tag{3}$$

and the effective torsional stiffness from the equation

$$GJ_{\text{eff}}/GJ_0 = 1 + [\textstyle\int_A \sigma_{yy} r^2 dA/GJ_0] \tag{4}$$

If Eqs. (1) and (2) are substituted into Eq. (3) and then into Eq. (4), with $r \simeq x$ and $dA = t\,dx$, one obtains

$$\frac{GJ_{\text{eff}}}{GJ_0} = 1 - \left(\frac{E}{G}\right)\left(\frac{c}{t_0}\right)^2\left(\frac{\alpha\Delta T}{4}\right)K \tag{5}$$

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* Reader in Aeronautical Engineering. Associate Fellow Member AIAA.

The parameter K is defined by

$$K = \frac{\left[\frac{(m+n+3)+R(n+3)}{(n+3)(m+n+3)}\right] - \left[\frac{(m+n+1)+R(n+1)}{(n+1)(m+n+1)}\right]\left[\frac{m+1}{m+1+R}\right]\left[\frac{m+3+3R}{3(m+3)}\right]}{\frac{1}{3}\left[1+\frac{3R}{m+1}+\frac{3R^2}{2m+1}+\frac{R^3}{3m+1}\right]} \quad (6)$$

where $R = \Delta t/t_0$. The most likely value is $R = -1$. The other extreme, for the flat plate, is $R = 0$, which is equivalent to the combination of values $R = -1$, $m = \infty$. Hence, only the values $R = -1$, $1 \leq m \leq \infty$ will be considered. Table 1 presents values of K for a range of values of m and n, and it is seen that K_{max} occurs in the range $1 \leq n \leq 3^{1/2}$ for $1 \leq m \leq \infty$.

If in a particular problem, the temperature rise distribution is given by

$$T = T_0 + \sum_n \Delta T_n |\beta|^n \quad (7)$$

then Eq. (5) becomes

$$\frac{GJ_{\text{eff}}}{GJ_0} = 1 - \left(\frac{E}{G}\right)\left(\frac{c}{t_0}\right)^2 \sum_n \left(\frac{\alpha \Delta T_n}{4}\right) K_n \quad (8)$$

No general conclusions can be drawn from Table 1 regarding the relative merits of differing section shapes because equal, external, thermal conditions would produce quite different temperature distributions in the various section shapes, i.e., ΔT and n are also dependent on the values of t_0, R, and m. Also, for a true comparative study it is reasonable to assume that the values of GJ_0 for the various sections should be equal. The relative values of t_0/c for this to be so are given in Table 2.

To determine the dependence of ΔT and n on m for general heating problems is beyond the scope of this note, but, by using a three-point correlation system, approximate values can be obtained for the case of zero thermal conductivity, instantaneous acceleration, and constant heat-transfer coefficient.

The temperature rise distribution is given by

$$T/[T_{AW^{(f)}} - T_{AW^{(0)}}] = 1 - \exp -[\lambda/(1-\beta^m)] \quad (9)$$

where $\lambda = 2h_f\tau/\rho_m C_m t_0$ and τ is time. The three unknown parameters in Eq. (2), i.e., T_0, ΔT, and n, can be determined by correlating Eqs. (2) and (9) at three points on a semichord. Three different correlation systems were considered in order to investigate the consequent effects on the results, viz., a) $\beta = 0, \frac{1}{3}, 1$; b) $\beta = 0, \frac{1}{2}, 1$; c) $\beta = 0, \frac{2}{3}, 1$. By this means one obtains

$$\Delta T/[T_{AW^{(f)}} - T_{AW^{(0)}}] = e^{-\lambda} \quad (10)$$

and Eq. (5) becomes

$$\frac{GJ_{\text{eff}}}{GJ_0} = 1 - \left(\frac{E}{G}\right)\left(\frac{c}{t_0}\right)^2 \alpha\,[T_{AW^{(f)}} - T_{AW^{(0)}}]\Lambda \quad (11)$$

where

$$\Lambda = Ke^{-\lambda}/4 \quad (12)$$

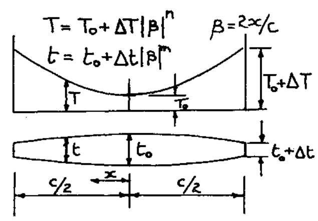
By assuming h_f to be constant chordwise, and using correlation system b, we obtain the relationship

$$(1 - 0.5^n)e^{-\lambda} = e^{-[\lambda/(1-0.5^n)]} \quad (13)$$

and, hence, n is determined as a function of λ, i.e., of time τ for a given value of m.

Figure 2 shows the variation of Λ with λ for $m = 1$ using the three correlation systems, together with the exact results of Ref. 1. The agreement is only fair and suggests that, although the use of simplified temperature distributions predicts the maximum value of Λ quite accurately, the predicted time for Λ_{max} is in some error.

Fig. 1 Solid wing showing assumed distribution of temperature.



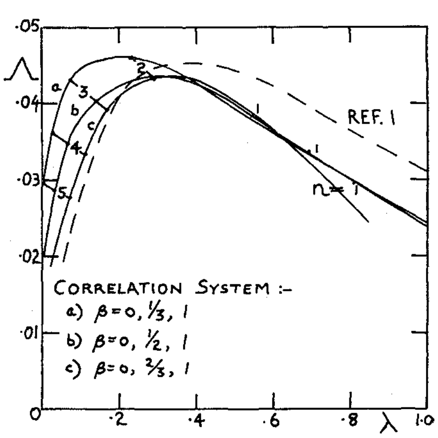


Fig. 2 Variation of Λ with $\lambda(m = 1)$.

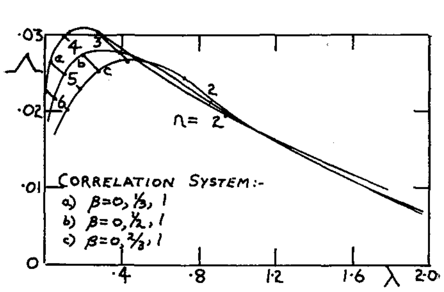


Fig. 3 Variation of Λ with $\lambda(m = 2)$.

Table 1 Variation of parameter K with m and n ($R = -1$)

n	$m = 1$	2	3	∞
0.5	0.229	0.182	0.172	0.190
1	0.267	0.219	0.210	0.250
1.1	0.267	0.220	0.211	0.255
1.2	0.265	0.220	0.212	0.260
1.5	0.256	0.216	0.210	0.267
$3^{1/2}$	0.247	0.210	0.205	0.268
2	0.233	0.200	0.197	0.267
3	0.186	0.164	0.164	0.250
4	0.148	0.132	0.136	0.229
5	0.119	0.109	0.112	0.208
6	0.098	0.091	0.095	0.190

Table 2 Relative values of t_0/c for equal values of J_0 in solid sections ($R = -1$)

$m = 1$	$m = 2$	$m = 3$	$m = \infty$
1.585	1.295	1.2	1

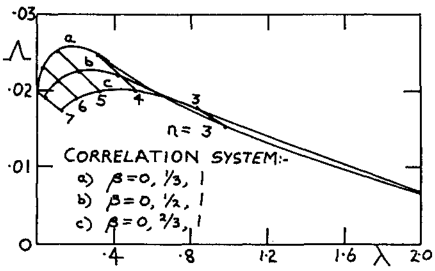


Fig. 4 Variation of Λ with $\lambda(m = 3)$.

Figures 3 and 4 show the corresponding results for $m = 2$ and $m = 3$, respectively. As in Fig. 2, the various correlation systems show wide differences up to the time of $\Lambda_{\max}$ but they agree well subsequently. The values of n corresponding to $\Lambda_{\max}$ are seen to be, approximately, 2, 3, and 4 for the cases $m = 1$, 2, and 3, whereas in Table 1 the corresponding values for $K_{\max}$ are $n = 1$, 1.1, and 1.2, respectively.

To summarize, the correlation system has been shown to have limited usefulness. The parameter K in Eq. (5) is not a reliable guide to the magnitude of the reduction in torsional stiffness since maximum values of K and Λ do not correspond, and the latter is a more useful parameter to assess the reduction, Eq. (11).

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Method of Belotserkovskii for Asymmetric Blunt-Body Flows

E. A. Brong*
General Electric Company, Philadelphia, Pa.
and
D. C. Leigh†
Princeton University, Princeton, N. J.

Introduction

There are several available methods of solution for symmetric supersonic blunt-body flows. Many of these methods are described by Hayes and Probstein.[1] On the other hand, for asymmetric flows there are very few available methods, even when the asymmetry does not introduce an additional independent variable as in the case of plane flow. One of the problems has been the determination of the stagnation streamline and, consequently, the value of the entropy on the body.[2] A popular hypothesis has been to assume that the streamline that wets the body in an asymmetric flow is the one that crosses the shock at right angles and, hence, has the maximum entropy in the shock layer. In particular, this hypothesis was made in applying the Belotserkovskii one-strip method to the problem of flow over an axisymmetric body at angle of attack. However, it is clear that the stagnation streamline ought to be determined by the method of solution and should not require an a priori hypothesis. Swigart[4, 5] has developed an inverse method‡ for symmetric bodies at small angles of attack which has this feature. In this method, no hypothesis is made regarding the position of the maximum-entropy streamline; its position in the flow is determined as part of the solution and it is in fact not the stagnation streamline.

The subject of this paper is the application of the Belotserkovskii one-strip method to plane asymmetric flows. This is a direct method in the sense that the shock and flow field are determined for a given body shape. For the one-strip method, it will be shown that the stagnation streamline is a straight line, and hence there are sufficient relations at the stagnation streamline so that an a priori hypothesis concerning the body entropy does not have to be made. This result also applies to the method of Refs. 2 and 3 for axisymmetric bodies at angle of attack so that in those works it would have been possible to determine the body entropy in the forementioned way.

Analysis

The method of Belotserkovskii is the application of Dorodnitsyn's method of integral relations to blunt-body flows and is described in Ref. 1 on pages 214–226. References to the papers of Dorodnitsyn and Belotserkovskii are given in Ref. 1. Briefly, the method consists of putting the governing fluid dynamic equations in "divergence form," assuming suitable linear approximations in the direction normal to the body surface, and then integrating in the normal direction from the body to the shock. There then results a set of ordinary differential equations in the direction of the body surface for certain dependent variables. Such a formulation has been made by Xerikos and Anderson[6] for symmetric bodies of general shape. We shall adopt their notation and equations in this paper.

The coordinate system and notation are shown in Fig. 1. The quantity s is the distance along the body surface, n is the distance normal to the surface, and $R(s)$ is the radius of curvature of the body at s. v_n is the component of flow in the n direction, and v_s is the component perpendicular to that direction. $\theta(s)$ is the body angle, $\chi(s)$ the shock angle, and $\delta(s)$ the shock-layer thickness. The subscript zero refers to quantities on the body and the subscript δ to quantities on the shock. The stagnation point on the body surface is the point at which $v_{s_0} = 0$; the subscript st refers to quantities at the stagnation point.

The linear approximations in the n direction employed by Xerikos and Anderson are

$$\begin{aligned} t &= t_0 + (n/\delta)(t_\delta - t_0) \\ z &= (n/\delta)z_\delta \\ G &= G_0 + (n/\delta)(G_\delta - G_0) \end{aligned} \tag{1}$$

where

$$\begin{aligned} t &\equiv (1 - V^2)^{1/(\gamma - 1)} \qquad V^2 = v_s^2 + v_n^2 \\ z &\equiv \rho v_s v_n \\ G &\equiv (1/R)\{\rho v_s^2 + [(\gamma - 1)/2\gamma]p\} \end{aligned} \tag{2}$$

Upon substitution of Eq. (1) in the governing fluid dynamic equations and integration in the n direction from the body to the shock, Xerikos and Anderson obtained the following simultaneous ordinary differential equations:

$$\frac{d\delta}{ds} = \frac{1 + (\delta/R)}{\tan(\theta + \chi)} \tag{3}$$

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* Engineer, Theoretical Aerodynamics, Re-Entry Systems Department, Missile and Space Division.

† Assistant Professor, Department of Aerospace and Mechanical Sciences. Member AIAA.

‡ In the inverse method the shock-wave shape, freestream conditions, and angle of incidence of the shock wave are specified, and the corresponding body and flow field are to be determined.